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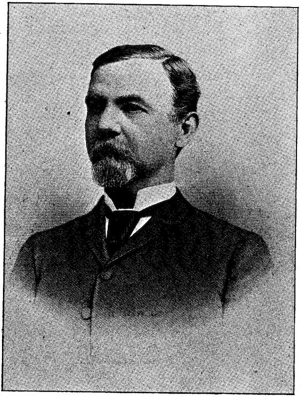
BIOGRAPHY.

COL. JAMES W. NICHOLSON.

Few persons in the South have acquired a more enviable reputation in their profession than Col. James W. Nicholson, the accomplished president and professor of Mathematics in the Louisiana State University and Agricultural and Mechanical College. This reputation is largely due to his mathematical attainments, which are fast becoming recognized throughout the scientific world and to the general ability and fidelity with which he has invariably discharged the various duties and trusts committed to him. Col. Nicholson was born in Macon county, Ala., June 16, 1844, but is to all intents a native of Louisiana, since his father, W. B. Nicholson, moved with his family to Claiborne parish, La., only six months after the birth of his son, and has continued to reside there ever since. The family connections in the northern portion of the state are very numerous on the maternal side, and include numerous names closely identified with the history of the State. Colonel Nicholson's early training was largely under the guidance of Prof. J. W. Boring, who prepared him for his admission to the freshman class of Homer college at the age of fourteen years. He was probably fairly well prepared for admission. He had inherited a robust constitution, in fact he is at the present time a model of physical health. His moral nature has been developed as is usual in Christian families and Christian communities along vigorous, healthful lines, and his intellectual advancement was fully up to the requirements of the college which he entered which were by no means of a low order. The college at this time was under the management of such men as Rev. Baxter Clegg, John W. Stacy, John B. Gretter and Professor Simmons, whose only view of an education was that it was intended to develop rather than to adorn men, and all of whose methods and processes were directed to this end. Under the guidance of such men as these a young man of Mr. Nicholson's bent and genius was sure to develop rapidly. Even



Col. James W. Nicholson.

during his preparatory course he had shown a strong preference for the study of mathematics, and a singular aptitude in the mastery of its principles. Details he cared but little about for the trend of his mind from the beginning was toward originality—generalizations. The usual methods of solution and a demonstration served to merely stimulate him to search for new and unusual methods of reaching the same results.

It is said that he never received any aid in the solution of a mathematical question, nor a single demerit for misconduct, while at college. His devotion to mathematical study did not, however, prevent his taking a fair stand in his classical studies. But his course of study was not to be completed at this time. To him as to other young men in our college in the year 1861, the sound of war's alarm was the call of a higher duty.

While still in his sixteenth year, and with but a single year more to study before graduation, he entered the confederate army in 1861 as a private in Company B, Twelfth Louisiana infantry, under Col. Thomas M. Scott, and served without interruption until the close of the war, and until surrendered at Greensboro, N. C., only twenty-three days less than four years from the date of his enlistment. It is doubtless due to his extreme youth (and his continued occupation of study even during the war) that he achieved no marked distinction among the brave fellows by whom he was surrounded, most of whom were by many years his seniors.

He was, however, elected second sergent of his company at Camp Green, Tenn., in the spring of 1862, in that capacity participated in many of the great battles of the war. Having returned home after the war, his first thought was for a profession in life, and at first his mind turned toward the law, and its study was commenced under the guidance of the late Judge W. B. Eagan, of Homer, La. This study was, however, of brief duration, for a vacancy occurring, or rather being, for it was in the period of reconstruction, in the chair of the professor of mathematics in Homer College, he was induced to fill it temporarily and continued to fill it for the term of two years. During these two years his old love for mathematical study which had never quite left him returned with full force, and finding teaching congenial, he resolved to make it his profession for life. About six miles from Homer, the home of his boyhood, in the little town of Arizona, an enterprise, most unusual in this section, had sprung into being. A company had been organized, a large cotton factory had been built, and a little village was springing into life around it. This seemed to young Nicholson a fine opening for a school, and he accordingly gave up his position in Homer college and started a new school in the fall with forty-five students, which number soon increased to 125 and the success of the new enterprise was assured. But his *alma mater* had not done with him yet.

In 1870 he was elected professor of mathematics in Homer college at a salary of $1,500 per annum. The school at Arizona, still his property, did not flourish, however, after he had left it, as it had done under his immediate management, and in the winter of 1872 he was compelled to give up the college

professorship and resume charge of the school in order to save his property. For the next five years he labored incessantly, the school surviving even the failure of the industrial venture upon which the life of the village of Arizona depended, meanwhile devoting himself with increasing ardor to the study of mathematics and consequently adding both to his knowledge and his reputation as a student in that branch of study in which in after years he was destined to acquire distinction.

During the summer of 1877 upon the re-organization of tne Louisiana State University and Agricultural and Mechanical College, which institutions had just been united into one, Professor Nicholson was elected to the chair of mathematics in this university, which stands at the head of the public school system of the state.

This position he has ever since continued to fill with credit to himself, the institution and the state. On Apri 13, 1883, Col. William Preston Johnson, then president of the Louisiana State University, having resigned the presidency of Tulane University at New Orleans, Professor Nicholson was elected president of the university in which he still continues to hold the chair of mathematics, discharging with great facility and efficiency the duties of both positions, either one of which would have taxed the energies of a less vigorous man. But his even temperament and firm constitution fit him to accomplish tasks beyond the powers of the average man, and the work of the president and the instructor alike prospered in his hands. This being a military school, the new rank of colonel was conferred upon him by the governor, a title which in this military age seems to be more generally recognized than those which testify to civic honors of greater worth, consequently, instead of Professor Nicholson or President Nicholson he is generally accosted as Colonel Nicholson. But whether as professor or president or Colonel, honor seems to sit lightly upon him, and he is the same genial companion, wise counselor and sympathetic friend. His administrative capacity is of high order. Full of expedients, his mind being always on his work and his heart in it, the school over which he presides is never allowed to stagnate, or to become disorderly. As in his mathematics, so here the solution of the problems may not be expected or usual, but they are sure to be prompt, vigorous, and effective. His magnetic nature draws hearts to him and there are few among either pupils or associates who do not esteem him as a friend.

The position of president of the State University, Col. Nicholson still (1894) occupies and has continued to occupy (with a brief interval) since his election in 1883, as stated above.

As Colonel Nicholson's reputation is chiefly that of a mathematician, this sketch would be imperfect without special mention of him in this particular, and of some of the contributions which he has made to mathematical science. As already intimated he has an independent, inventive and progressive mind, always more disposed to invent new methods than to passively follow old ones, and as a consequence he has extended in several lines of first importance the scope of mathematical inquiry, and his merit has been recog-

onized by the foremost mathematicians of the world. He is a member of the London Mathematical Society (England), and also of the Mathematical Society, of New York. A partial list of his mathematical works, formulas, etc., is as follows:

He has been a contributor to most of the mathematical journals of the country. He is the discoverer of the following singular value of π: $\pi=2\div[(1-1)^{\frac{1}{2}}(1-1)^{-\frac{1}{2}}]$; and also of the following: $\cos\phi=\frac{(-1)^{\frac{\phi}{\pi}}+(-1)^{-\frac{\phi}{\pi}}}{2}$ and $\sin\phi=\frac{(-1)^{\frac{\phi}{\pi}}-(-1)^{-\frac{\phi}{\pi}}}{2\sqrt{-1}}$.

A series of arithmetics and an elementary algebra, adopted and in exclusive use in the public schools of Louisiana. Also a treatise on "Isoperimetrical Geometry" in 1869, one on the Calculus of Finite Differences" in 1871, one on Directed Quantities in 1885, etc. His method of developing the last named subject, which is a new method of imaginaries, is entirely different from that of Argand or Hamilton, and the author thinks of publishing the same at an early day. In 1868 he published a pamphlet on the "Trigonometrical Circle," a formula which he devised for expressing the relation between the sides and functions of the angles of right angled triangles, which has been incorporated into some of the standard works on trigonometry, and taught in some of the best colleges and universities in this country and Europe. In 1880 he published a pamphlet entitled "A New and Complete Demonstration of the Bonomial Theorem" which has received the highest commendation from mathematicians. He published a pamphlet on the "Multisector," an instrument which he has invented for dividing an angle into any number of equal parts. A meritorious paper of his on "a simple and direct method of separating the roots of ordinary equations" was read before the Mathematical Society, of New York, May 7, 1892. His last great contribution to mathematics is "A Direct and General Method of finding the Real Roots of Numerical Equations to any Degree of Accuracy."

He has just finished the manuscript of a work on the Differential and Integral Calculus on a *new plan or theory*, which will be printed and ready for use in a few months.

The degree of LL. D. was conferred on Col. Nicholson by the A. and M. College, Ala., in 1893.

Col. Nicholson married in Clairborne parish, July 30, 1876, Miss Sallie D. Baker, a native of that parish, the daughter of Capt. James C. Baker, a native of Georgia, a captain in the confederate army. By this marriage Colonel Nicholson has five children: Gordon, Lilburne, Malcolm Dudley, Wilbur Fenner, and Annie. Both Colonel Nicholson and his wife are members of the Methodist church. Col. Nicholson is a liberal patron of art, and nothing affords him more pleasure than to study the faces and lives of the masters among men, for which he is firm in the philanthropy of his religion, he believes man to be the crowning work of creation, whose destiny is worthy

the study and toil of ages. In all this he holds the noblest of all possessions, a brave, intelligent, and trusting wife, whose sympathy and encouragement is a constant incentive to him to work on, to penetrate still deeper in the hidden mysteries of his laborious science. He is tall, being five feet eleven inches in height, and weighs 185 pounds. He has light grayish blue eyes and a face which leaves the impression of power and capacity.

[Erratum.--Page 185, beginning of 16th line from the top of page, insert "to accept the presidency".]

SOME NOVEL AND INTERESTING FORMULAS.

By J. W. NICHOLSON, A. M., LL. D., Member of the London, and New York Mathematical Societies, and President and Professor of Mathematics, Louisiana State University, Baton Rouge, Louisiana.

These formulas are given without demonstration, thinking that their deduction would occupy more space than they probably deserve.

$$(2a+2b)^2+a^2+b^2=(2a+b)^2+(a+2b)^2 \ldots\ldots (1).$$

This is a simple formula for finding three square numbers whose sum is equal to the sum of two squares. Thus, for $a=5$, $b=3$, we have

$$16^2+5^2+3^2=13^2+11^2 \ldots\ldots (2).$$

$$(3a+3b)^n+(2a+4b)^n+a^n+b^n=(3a+4b)^n+(a+3b)^n+(2a+b)^n \ldots\ldots (3),$$

where $n=3$, 2 or 1.

Thus, for $a=5$, $b=3$, we have $24^n+22^n+5^n+3^n=27^n+14^n+13^n \ldots (4)$, where $n=3$, 2 or 1.

$$(5a+10b)^n+(4a+11b)^n+(3a+5b)^n+(2a+8b)^n+(3a+3b)^n+(2a+6b)^n +a^n+b^n=(5a+11b)^n+(4a+6b)^n+(3a+10b)^n+(3a+8b)^n+(a+5b)^n+(2a+3b)^n +(2a+b)^n \ldots (5),$$

where $n=5$, 4, 3, 2 or 1.

Thus for $a=5$, $b=2$, we have

$$45^n+42^n+26^n+25^n+22^n+21^n+5^n+2^n=47^n+35^n+32^n+31^n+16^n+15^n +12^n \ldots (6),$$

where $n=5$, 4, 3, 2 or 1.

In (5) for $a=8$, $b=3$, we find $15^n+10^n+9^n+6^n=14^n+13^n+7^n+3^n+2^n+1^n \ldots (7)$, where $n=5$, 3 or 1.

$$(a+32)^n+(a+24)^n+(a+18)^n+(a+10)^n+(a+4)^n+(a-4)^n+(a-10)^n +(a-18)^n+(a-24)^n+(a-32)^n=(a+30)^n+(a+28)^n+(a+16)^n+(a+8)+{}^n(a+6)^n +(a-6)^n+(a-8)^n+(a-16)^n+(a-28)^n+(a-30)^n \ldots (8),$$

where $n=5$, 4, 3, 2 or 1.

In (8) by making $a=7$, we find

$$39^n+31^n+21^n+9^n=37^n+35^n+15^n+13^n \ldots (9),$$

where $n=5$, 3 or 1.

$$j=\underline{|n}=n^n-n(n-1)^n+\frac{n(n-1)}{2}(n-2)^n-\frac{n(n-1)(n-2)}{2.3}(n-3)^n+\&c. \ldots (10),$$

where n is any positive integer.

Thus, for $n=5$, we have

$$1.2.3.4.5=5^5-5(4)^5+10(3)^5-10(2)^5+5(1)^5\ldots(11).$$

Some new and interesting properties of prime numbers.

If $n+1$ is a prime number, then

$$(a+nb)^x+[a+(n-1)b]^x+[a+(n-2)b]^x+\ldots.(a+b)^x+a^x=m(n+1)\ldots.(12),$$

where m is an integer and x any integer less than n.

In (12) for $a=0$ and b=1, we have

$$n^x+(n-1)^x+(n-2)^x\ldots.2^x+1=m(n+1)\ldots.(13).$$

Thus, 11 will exactly divide

$$10^x+9^x+8^x+7^x+6^x+5^x+4^x+3^x+2^x+1\ldots.(14) \text{ where } x=9,8,\ 7,\ 6,\ 5,\ 4,\ 3,\ 2,1.$$

The converse of formulas (12) or (13) is not always true, but the following are true only when $n+1$ is a prime number.

$$(a+n)^n+(a+n-1)^n+(a+n-2)^n+\ldots.a^n+1=m(n+1)\ldots.(15).$$

Making $a=0$, we have

$$n^n+(n-1)^n+(n-2)^n\ldots.1^n+1=m(n+1)\ldots.(16).$$

That is, S_n+1 is divisible by $n+1$ when it is a prime number and only when it is prime. So far as I know this furnishes an entirely *new criterion* of *prime numbers.*

NOTE. The preceding formulas are taken from a paper, by the author, on "The nth power of any number expressed as the sum of the nth powers of other numbers, n being any positive integer;" which was read before the New York Mathematical Society, Dec. 3d, 1892.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

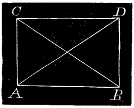
BY GEORGE BRUCE HALSTED, A. M., (Princeton) Ph. D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

CHAPTER SECOND.

THE FIRST TREATISE ON NON-EUCLIDEAN GEOMETRY.

[Continued from the May Number.]

PROPOSITION I. *If two equal straights* [sects] (*fig 1.*) *AC, BD, make with the straight AB angles equal toward the same parts: I say that the angles at the join CD will be mutually equal.*

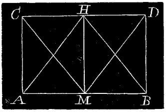


PROOF. Join *AD, CB*. Then consider the triangles *CAB, DBA*. It follows (Eu. I. 4.) that the bases *CB, AD* will be equal.

Then consider the triangles *ACD, BDC*. It

follows (Eu. I. 8.) that the angles ACD, BDC will be equal. Quod erate demonstrandum.

PROPOSITION II. *Retaining the uniform quadrilateral $ABCD$, bisect the sides AB, CD (fig. 2) in the points M and H. I say the angles at the join MH will then be right.*



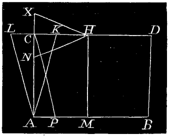
PROOF. Join AH, BH, and likewise CM, DM.

Because in this quadrilateral the angles A, and B are taken equal and likewise (from the preceding proposition) the angles C, and D are equal; it follows (Eu. I. 4.) (noting the equality of the sides) that in the triangles CAM, DBM, the bases CM, DM will be equal; and likewise, in the triangles ACH, BDH, the bases AH, BH.

Therefore; comparing the triangles CHM, DHM, and in turn the triangles AMH, BMH; it follows (Eu. I. 8.) that we have mutually equal, and therefore right the angles at the points M, and H.

Quod erat demonstrandum.

PROPOSITION III. *If two equal straights* [sects] (*fig* 3.) AC, BD *stand perpendicular to any straight AB: I say the join CD will be equal, or less, or greater than that AB, according as the angles at the same CD are right, or obtuse, or acute.*



Proof of the First Part. Each angle C, and D, being right; suppose, if it were possible, either one of those, as DC, greater than the other BA.

Take in DC the piece DK equal to BA, and join AK. Since therefore on BD stand perpendicular the equal straights BA, DK, the angles BAK, DKA will be equal (P. I.). But this is absurd; since the angle BAK is by construction less than the assumed right angle BAC; and the angle DKA is by construction external,and therefore (Eu. I. 16.) greater than the internal and opposite DCA, which is supposed right. Therefore neither of the aforesaid straights,DC, BA, is greater than the other,whilst the angles at the join CD are right; and therefore they are mutually equal.

Quod erat primo loco demonstrandum.

PROOF OF THE SECOND PART. But if the angles at the join CD are obtuse bisect AB, and CD, in the points M, and H, and, join MH.

Since therefore on the straight MH stand perpendicular (P. II.) the two straights AM, CH, and at the join AC is a right angle at A, the straight CH will not be (P. I.) equal to this AM, since a right angle is lacking at C.

But neither will it be greater: otherwise in HC the piece KH being assumed equal to this AM, the angles at the join AK will be (P. I.) equal.

But this is absurd, as above. For the angle MAK is less than a right; and the angle HKA is (Eu. I. 16.) greater than an obtuse, such as the internal. and opposite HCA is supposed.

It remains therefore, that CH, whilst the angles at the join CD are taken obtuse, is less than this AM; and therefore CD double the former is less than AB double the latter. Quod erat secondo loco demonstrandum.

PROOF OF THE THIRD PART. Finally however, if the angles at the join CD are acute, MH being constructed as before perpendicular (P. II.), we proceed thus. Since on the straight MH stand perpendicular two straights AM, CH, and at the join AC is a right angle at A, (as above) the straight CH will not be equal to this AM since the angle at C is not right. But neither will it be less: otherwise; if in HC produced HL is taken equal to this AM; the angles at the join AL will be (as above) equal.

But this is absurd. For the angle MAL is by construction greater than the assumed right MAC; and the angle HLA is by construction internal, and opposite, and therefore less than (Eu. I. 16.) the external HCA, which is assumed acute.

It remains therefore, that CH, whilst the angles at the join CD are acute, is greater than this AM, and therefore CD the double of the former is greater than AB the double of the latter. Quod erat tertio loco demonstrandum.

Therefore it is established that the join CD will be equal, or less, or greater than this AB, according as the angles at the same CD are right, or obtuse, or acute. Quae erant demonstranda.

COROLLARY I. Hence in every quadrilateral containing assuredly three right angles, and one obtuse, or acute, the sides adjacent to this oblique angle are less than the opposite sides, if this angle is obtuse, but greater if it is acute.

For this has just now been demonstrated of the side CH relatively to the opposite side AM; in the same way it is demonstrated of the side AC relatively to the opposite side MH. For since the straights AC, MH, are perpendicular to this AM, they cannot (P. I.) be mutually equal, on account of the unequal angles at the join CH.

But neither (in the hypothesis of an obtuse angle at C) can a certain AN, a piece of this AC, than which certainly the aforesaid AC is greater, be equal to this MH: otherwise (P. I.) the angles at the join HN would be equal; which is absurd, as above.

Again however (in the hypothesis of an acute angle at this point C) if you take a certain AX, assumed on AC produced, than which certainly the just mentioned AC is less, equal to this MH; now by this same title the angles at HX will be equal; which assuredly is absurd in the same way, as above.

It remains therefore, that indeed in the hypothesis of an obtuse angle at this point C, the side AC is less than the opposite side MH; but in the hypothesis of an acute angle is greater than it. Quod erat intentum.

COROLLARY II. But by much more will CH be greater than any piece of this AM, as for instance PM, with which of course the join CP makes an angle still more acute with this CH towards the parts of the point H, and obtuse (Eu. I. 16.) with this PM towards the parts of the point M.

COROLLARY III. Again it abides that all things aforesaid equally result, whether the assumed perpendiculars AC, and BD are of some length

fixed by us, or are, or are supposed infinitesimal.

This indeed ought opportunely to be noted in remaining subsequent propositions.

PROPOSITION IV. *But inversely (the figure of the preceding proposition remaining) the angles at the join CD will be right, or obtuse, or acute, according as the straight CD is equal, or less, or greater than the opposite AB.*

PROOF. For if the straight CD is equal to the opposite AB, and nevertheless the angles at it are either obtuse, or acute; now these such angles prove it (P. III.) not equal, but less, or greater that the opposite AB; which is absurd against the hypothesis.

The same uniformly avails in regard to the remaining cases. It holds therefore that the angles at the join CD are either right, or obtuse, or acute, according as the straight CD is equal, or less, or greater than the opposite AB. Quod erat demonstrandum.

DEFINITIONS. Since (P. I.) the straight joining the extremities of equal perpendiculars standing upon the same straight (which we call base), makes equal angles with these perpendiculars; therefore there are three hypotheses to be distinguished about the species of these angles. And the first indeed I will call hypothesis of right angle; the second however, and the third I will call hypothesis of obtuse angle, and hypothesis of acute angle.

ARITHMETIC.

Conducted by B.F.FINKEL,Kidder, Missouri. All Contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

16. Proposed by EARL D. WEST, West Middleburg, Logan County, Ohio.

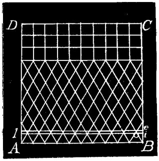
How many stakes can be driven down upon a space 15 feet square allowing no two to be nearer each other than $1\frac{1}{2}$ feet, and how many allowing no two to be nearer than $1\frac{1}{4}$ feet?

Solution by B. F. FINKEL, A. M., Professor of Mathematics, Kidder Institute, Kidder, Missouri.

(a) 1. Since the least distance from one stake to another is $1\frac{1}{2}$ ft., the number of $1\frac{1}{2}$ ft. spaces in the base line AB is 15 ft. $\div 1\frac{1}{2}$ft. or 10. Hence, we can place 11 stakes on the base line AB, and, by *square arrangement*, we can place on the square $ABCD$ 11 rows with 11 stakes in a row, in all 11×11 stakes or 121 stakes.

2. By *quincunx arrangement*, we can place 11 stakes on the base line AB and over these, as vertices of equilateral triangles 10 stakes. Now the width Bi of the strip $ABil$ is $\sqrt{oB^2-oi^2}=\sqrt{(1\frac{1}{2})^2-(\frac{3}{4})^2}=\frac{3}{4}\sqrt{3}$ ft.$=1.2990381+$ ft. Hence, the width of the strip le is 1.5 ft.$-1.2990381+$ ft.$=2009619-$ ft.

Therefore, for every strip $1\frac{1}{2}$ ft. wide, there is a gain, by the *quincunx arrangement*, of .2009619 ft. Now if we gain .2009619 ft. for 1 strip $1\frac{1}{2}$ ft. wide, how many $1\frac{1}{2}$ ft. strips must be taken to gain $1\frac{1}{2}$ ft.? By Proportion, we have .2009619 ft. : $1\frac{1}{2}$ ft. :: 1 strip $1\frac{1}{2}$ ft. wide : (?=7 strips $1\frac{1}{2}$ ft. wide). Hence, in 7 strips $1\frac{1}{2}$ ft wide, we have 8 strips 1.2990381 ft. wide, and therefore have 9 rows of stakes, 5 rows of 11 in a row and 4 rows of 10 in a row—in all, 95 stakes.



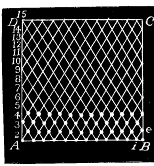
It is seen from the figure that there are 3 strips left. On these three strips we can make 3 rows of 11 stakes in a row, or 33 stakes on the remaining strips using the *square arrangement*. Hence, we can place 95 stakes +33 stakes or 128 stakes on the square.

(*b*) 1. When the least distance from one stake to another is $1\frac{1}{4}$ ft., the number of $1\frac{1}{4}$ ft. spaces on the base line AB is 15 ft. $\div 1\frac{1}{4}$ ft. or 12. Hence, we can place 13 stakes on the base line AB, and, by *square arrangement*, we can place on the square 13 rows of 13 stakes in each row—in all, 13×13 stakes or 169 stakes.

2. By *quincunx arrangement*, we can place 13 stakes on the base line and over these, as vertices of equilateral triangles, 12 stakes.

Following the same method as (2) of (*a*), we find we can have 5 rows of 13 stakes in each row or 65 stakes and 4 rows of 12 stakes in each row or 48 stakes—in all 113 stakes. Now there will be 5 strips left, on which, by *square arrangement*, we can have 5 rows of 13 stakes in each row or 65 stakes. Hence, by the *square and quincunx* arrangements combined, we can place 113 stakes +65 stakes or 178 stakes on the square.

3. If we could have 15 rows of 12 stakes in each row, instead of 14 rows, 10 rows having 13 stakes each and 4 rows having 12 stakes each, we would have 180 stakes. If such an arrangement is possible, we must place the stakes in the rows more than $1\frac{1}{4}$ ft. apart and thus allow the rows to come closer together. Since we are to have 15 rows, there will be 14 strips, each 15 ft. $\div 14$ or $\frac{15}{14}$ ft. wide. By placing the first row of 12 stakes on the base line AB (Fig. 2), we have $11\frac{1}{2}$ spaces (the least number of spaces) each 15 ft. $\div 11\frac{1}{2}$ or $\frac{30}{23}$ ft. In Fig. 2, $Bi=\frac{1}{2}$ of $\frac{30}{23}$ ft. $=\frac{15}{23}$ ft. and $Be=\frac{15}{14}$ ft. $\therefore\ ei=\sqrt{Bi^2+Be^2}=\sqrt{(\frac{15}{23})^2+(\frac{15}{14})^2}$ $=15\sqrt{\dfrac{14^2+23^2}{23^2\times14^2}}=\dfrac{75}{322}\sqrt{29}=1.2512+$ ft. Since this is more than the allowable distance, 1.25 ft., the arrangement is possible and we can place 180 stakes on the square, by arranging them as shown in Fig. 2.



This problem was also solved by *H. W. HOLYCROSS*, *G. B. M. ZERR*, *M. A. GRUBER*, *H. C. WHITAKER*, and *I. L. BEVERAGE*.

Mr. Holycross and Mr. Beverage get the same answers as above, but Mr. Beverage says that the 180 stakes are placed in a manner similar to the 128 stakes. This is not true. The other results differ from the above and from each other.

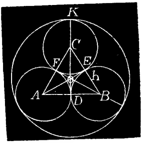
17. **Proposed by H W. HOLYCROSS, Superintendent of Schools, Pottersburg, Union County, Ohio.**

A gentleman owns a circular farm, and if three circles of equal area and as large as possible be drawn within it, the circular area in the center of the farm will contain one acre; what is the area of the circular farm?

Solution by SETH PRATT, C. E., Assyria, Michigan; Professor G. B M. ZERR, Principal of High School, Staunton, Virginia; A. H. BELL, Hillsboro, Illinois; and J. W. Watson, Middle Creek, Ohio.

Let $AD=BD=CE=r$, $OK=R$, and area $FDE=1A.=160$ sq. rd. $=a$. Then $AB=AC=BC=2r$, $CD=\sqrt{(AC^2-AD^2)}=\sqrt{(4r^2-r^2)}=r\sqrt{3}$, and area of $\triangle ABC=\frac{1}{2}(AB\times CD)=\frac{1}{2}(2r\times r\sqrt{3})=r^2\sqrt{3}$.

The area of the circular sector $DAF=\frac{1}{6}$ of circle whose center is $A=\frac{1}{6}\pi r^2$. Hence, the area of the three sectors within the triangle $ABC=\frac{1}{2}\pi r^2$.



Hence, the area $FDE=r^2\sqrt{3}-\frac{1}{2}\pi r^2=\frac{1}{2}r^2(2\sqrt{3}-\pi)=a$, whence $r=\sqrt{\frac{2a}{2\sqrt{3}-\pi}}$. Now $CD=r\sqrt{3}=\sqrt{3}\sqrt{\frac{2a}{2\sqrt{3}-\pi}}$ and $OC=\frac{2}{3}CD=\frac{2}{3}\sqrt{3}\sqrt{\frac{2a}{2\sqrt{3}-\pi}}$.

Hence, $R=OK=OC+CK=\frac{2}{3}\sqrt{3}\sqrt{\frac{2a}{2\sqrt{3}-\pi}}+\sqrt{\frac{2a}{2\sqrt{3}-\pi}}=\left(\frac{2\sqrt{3}+3}{3}\right)\sqrt{\frac{2a}{2\sqrt{3}-\pi}}$. ∴ Area of the circle whose center is $O=\pi R^2=\frac{2}{3}\pi\left(\frac{7+4\sqrt{3}}{2\sqrt{3}-\pi}\right)a=90.4534+$A.

This problem was also solved by *JOHN T. FAIRCHILD*, *J. F. W. SCHEFFER*, *M. A. GRUBER*, *H. C. WHITAKER*, *H. W. HOLYCROSS*, *I. L. BEVERAGE*, and *P. S. BERG*.

18. **Proposed by L. B. HAYWARD, Superintendent of Schools, Bingham, Ohio.**

In a circle whose radius is 6, find the area of the part between parallel chords whose lengths are 8 and 10, both being on the same side of the center.

Solution by H. W. HOLYCROSS, Superintendent of Schools, Pottersburg, Ohio; P. S. BERG, Apple Creek, Ohio; I. L. BEVERAGE, Monterey, Virginia; Professor G. B. M. ZERR, Principal of High School, Staunton, Virginia.

$h'=6-\sqrt{6^2-4^2}=1.528$, height of smaller segment.

$h=6-\sqrt{6^2-5^2}=2.683$, height of larger segment.

Area of larger segment $=\frac{h^3}{2b}+\frac{2hb}{3}=454-\frac{371}{3}\sqrt{11}=43.8595$.

Area of smaller segment$=\frac{h'^3}{2b'}+\frac{2h'b'}{3}=68-80\sqrt{5}=8.37173+$

∴ Required area $=43.8595-8.37173=35.48777$.

This problem was also solved by *H. C. WHITAKER*, *W. F. BRADBURY*, *JOHN T. FAIRCHILD*, *J. F. W. SCHEFFER*, and *J. W. WATSON*.

PROBLEMS.

25. **Proposed by L. B. HAYWARD, Superintendent of Schools, Bingham, Ohio.**

A company engaged an agent to do business for one month at a salary of \$25, giving him goods amounting to \$57·54 and \$32.17 in cash to start with. The agent bought during the month, goods amounting to \$59.91. At the end of the month the

goods on hand amounted to \$31.67, and the amount of sales for the month was \$102.97; what was the balance of acc't?

26. **Proposed by E. S. LOOMIS, A. M., Ph. D., Professor of Mathematics, Baldwin University, Berea, Ohio.**

You say, "While treating of the pronunciation of those who minister in public, two other words occur to me which are commonly mangled by our clergy. One of *these* (*A*) is 'covetous,' and its substantive 'covetiousness'. I hope some who read *these lines* will be induced to leave off pronouncing *them* (*B*) 'covetius' and 'coviousness'. I can assure *them* (*C*) that when *they* (*D*) do thus call *them* (*E*) one at least of *their* (*F*) hearers has his appreciation of *their* (*G*) teaching disturbed."-

The problem now is, in how many ways can this above quotation be (read or) understood, by supposing various antecedents to the pronouns as per table.

The pronoun	Nouns to which they apply.	No. of nouns.
(*A*) these	Words or clergy.	2
(*B*) them	Words, clergy, readers, or lines.	4
(*C*) them	Words, clergy, readers, or lines.	4
(*D*) they	Words, clergy, readers, or lines.	4
(*E*) them	Words, clergy, readers, or lines	4
(*F*) their	Words, clergy, readers, or lines.	4
(*G*) their	Words, clergy, readers, lines or hearers.	5

Solutions to these problems should be received on or before August 1st.

ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

14. **Proposed by SYLVESTER ROBINS, North Branch Depot, New Jersey.**

In copying the following example, the class lost the coefficient of x in the equation:

$$\sqrt{x}+\sqrt[3]{y}=x\ldots.(1),$$
$$\sqrt{x^3}+\sqrt[3]{y^2}=(m)\,x\ldots.(2).$$

and set themselves to finding values for m, which would allow rational values to x and y.

Solution by Hon. JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

Transposing (1) and squaring, we have

$\sqrt[3]{y^2}=x^2+x-2x\sqrt{x}=$(from (2)) $mx-x\sqrt{x}$, or reducing $x-\sqrt{x}=m-1\ldots.(3)$.

It follows from (1) that if $\sqrt{x}$ is rational, y will be rational also.

From (3) $\sqrt{x}=\dfrac{1\pm\sqrt{4m-3}}{2}$. Let $4m-3=p^2$, then $m=\dfrac{p^2+3}{4}$, and

$\sqrt{x}=\dfrac{p+1}{2}$, in which p may be any number. If $p=1$, $m=1$, $\sqrt{x}=1$, $x=1$, and $y=0$. Take $p=3$, $m=3$, $\sqrt{x}=2$, $x=4$ and $y=8$. To obtain integral results, take $p=2q+1$; then $m=q^2+q+1$, $\sqrt{x}=q+1$, $x=(q+1)^2$ and $y=[(q+1)^2-(q+1)]^3$, in which q may be any integral number.

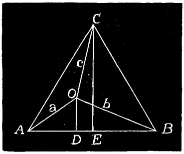
Also solved by *A. H. BELL*, *H. W. DRAUGHON*, *R. H. YOUNG*, *G. B. M. ZERR*, and the *PROPOSER*.

15. Proposed by SETH PRATT. C. E., Assyria, Michigan.

From a point in an equilateral triangle, the distances to the angles are, respectively, 20, 28, and 31 rods. Required a side of the triangle.

Solution by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

To establish a general formula, we denote the three given lines by a, b, c, and the required side by x.



Let the perpendicular $OD=z$, $AD=y$, $CE=\frac{x}{2}\sqrt{3}$, $AE=\frac{x}{2}$. Now, we obviously have the three equations: $y^2+z^2=a^2 \ldots .(1)$, $(x-y)^2+z^2=b^2 \ldots .(2)$, and $\left(\frac{x}{2}\sqrt{3}-z\right)^2+\left(\frac{x}{2}-y\right)^2=c^2 \ldots .(3)$.

Substituting the value of $y^2+z^2=a^2$ in (2), we get $x^2-2xy=b^2-a^2$, whence, $y=\frac{a^2-b^2+x^2}{2x} \ldots .(4)$, and the value of $x^2+y^2+z^2$ from (2), in (3) and also (4) in (3), we obtain after a few easy reductions, $z=\frac{a^2+b^2-2c^2+x^2}{2x\sqrt{3}} \ldots .(5)$. Substituting (4) and (5) in (1), we have without trouble the quadratic, $x^4-(a^2+b^2+c^2)x^2=a^2b^2+a^2c^2+b^2c^2-a^4-b^4-c^4$, whence

$x^2=\frac{1}{2}[a^2+b^2+c^2+\sqrt{3}\sqrt{2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4}]$ or if we denote the area of the triangle whose sides are a, b, c, by $\triangle$, we obtain the elegant expression, $x^2=\frac{1}{2}[a^2+b^2+c^2+4\triangle\sqrt{3}]$.

Substituting numerical values, we find $x^2=\frac{3}{2}[715+\sqrt{401557}]=44.97+$. The minus value of the radical must be rejected for the case that O is without the triangle.

Also solved in various ways by *P. S. BERG, J. W. WATSON, H. C. WHITAKER, R. H. YOUNG, G. B. ZERR, L. B. FRAKER*, and the *PROPOSER*.

16. Proposed by COLMAN BANCROFT, Professor of Mathematics, Hiram College, Hiram, Ohio.

A traveler whose speed constantly increases in a geometrical progression passes A at 2 o'clock, B at 3:30, C at 4:30, and D at 6:18. At B he is moving at the rate of 12 miles per hour, and at C 18 miles. Find his rate at A and D, and the distance from A to each of the points B, C, and D.

Solution by H. C. WHITAKER, B. S., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

Let a denote the rate at A, d the rate at D, r the rate of acceleration per hour. At the end of time t after 2 o'clock, the rate will be $a\left(1+\frac{r}{x}\right)^{tx}$, x being the number of times per hour the acceleration is added; this when x is infinite equals ae^{tr}; hence the equations $ae^{1.5r}=12$, $ae^{2.5r}=18$, $ae^{4.3r}=d$.

The distance $=\int_0^t ae^{tr}dt=\frac{a}{r}\left(e^{tr}-1\right)$.

Substituting for e its value 2.71828 and solving these equations,

$r=.405464$ miles per hour, $a=6.532$ miles per hour, $d=47.0148$ miles per hour, $AB=13.4857$ miles, $AC=28.29$ miles, and $AD=76.17$ miles.

Similarly solved by *Professor G. B. M. ZERR*, and solved under a different interpretation by *P. S. BERG, J. F. W. SCHEFFER*, and *J. W. WATSON*.

17. Proposed by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

A sum of P dollars is loaned at r per cent. interest. At the end of the first year a payment of x dollars is made; and at the end of each following year the payment is made greater by m per cent. than the preceding year. If the sum is paid in n payments, find x.

Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University P. O., Mississippi.

Let $R=1+r$, $M=1+m$.

Amt. owed after 1st. payment $=PR-x$.

.. .. .. 2nd. .. $=(PR-x)R-xM=PR^2-xR-xM$.

.. .. .. 3rd. .. $=PR^3-xR^2-xMR-xM^2$.

.. .. .. 4th. .. $=PR^4-xR^3-xMR^2-xM^2R-xM^3$.

....

.. .. .. nth. .. $=PR^n-x(R^{n-1}+MR^{n-2}+M^2R^{n-3}+\dots M^{n-2}R+M^{n-1})$.

If n payments cancel the debt, this $=0$. The quantity in the parenthesis is a geometrical progression, the first term being R^{n-1}, the ratio $\frac{M}{R}$, the number of terms n.

$$\text{The sum is } \frac{R^{n-1}\left[\left(\frac{M}{R}\right)^n-1\right]}{\frac{M}{R}-1}=\frac{M^n-R^n}{M-R}.$$

$$\therefore \frac{M^n-R^n}{M-R}x=PR^n,$$

$$\text{and } x=\frac{M-R}{M^n-R^n}PR^n=\frac{(m-r)(1+r)^n}{(1+m)^n-(1+r)^n}P.$$

Also solved by *Professors SCHEFFER, WHITAKER*, and *ZERR*.

18. Proposed by WILLIAM E. HEAL, Member of the London Mathematical Society, Marion, Indiana.

Two railroad trains, lengths m and n, meet at a siding, length l. How shall the trains pass if $l<m<n$?

Solution by W. H. CRALLE, Department of Mathematics, Hogsett Academy, Danville, Kentucky.

Divide train m into sections l or less in length, and with its engine pull its first section on siding; then let engine of n take its train beyond the switch, attach to next section of m and back it till rear of n clears the switch; then let engine of m pull its first section above the switch; then let n back on to switch and beyond, leaving second section of m on switch; next, let engine of n pull its train again beyond switch; then engine of m backs first section into switch and pulls off the second section. This operation is repeated $\frac{m}{l}$ times

when all the sections of m will have been passed up, and the trains may proceed.

Variously solved by *P. S. BERG, C. W. M. BLACK, H. W. DROUGHON, H. C. WHITAKER*, and *G. B. M. ZERR.*

PROBLEMS.

23. Proposed by ROBERT J. ALEY, A. M., Professor of Mathematics in Indiania University, Bloomington, Indiana.

Sum to n terms the following series: $11+25+45+71+103+\ldots.$

24. Proposed by ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Find x and y from the equations,

$$x^2+2xy^2-2xy=4y^3,$$
$$y^4+2x^3y^2-2xy^2=4x^4.$$

25. Proposed by P. C. CULLEN, Meade, Nebraska.

A man and a boy get n dollars for digging potatoes, the man can dig them as fast as the boy can pull the vines, but the man can pull vines m times as fast as the boy can dig them. Divide the money.

Solutions to these problems should be received on or before August 1st.

GEOMETRY.

Conducted by B.F.FINKEL,Kidder, Missouri. All Contributions to this department should be sent to him.

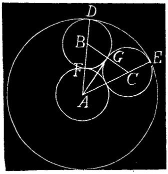
SOLUTIONS TO PROBLEMS.

11. Proposed by MISS LECTA MILLER, B. L., Professor of Natural Science and Art, Kidder Institute, Kidder, Missouri.

A gentleman's residence is at the centre of his circular farm containing $a=900$ acres. He gives to each of his $m=7$ children an equal circular farm as large as can be made within the original farm; and he retains as large a circular farm of which his residence is the centre, as can be made after the distribution. Required the area of the farms made.

Solution by Professor P. H. PHILBRICK, C. E., Lake Charles, Louisiana; and Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let A be the centre of the farm, B, C, the centres of two of the m farms for the children. Let $BG=GC=x$.



Then $AB=r-x$ where r is the radius of the whole farm. $AF=r-2x$.

Then $BG:AB=\sin BAG:1$,

$$\therefore\quad x=(r-x)\sin\frac{\pi}{m}.\quad\therefore\ x=\frac{r\sin\frac{\pi}{m}}{1+\sin\frac{\pi}{m}}.$$

$$r-2x=\frac{r\left(1-\sin\frac{\pi}{m}\right)}{1+\sin\frac{\pi}{m}};\ \pi x^2=\pi r^2\left\{\frac{\sin\frac{\pi}{m}}{1+\sin\frac{\pi}{m}}\right\}^2=\frac{a\left(\sin\frac{\pi}{m}\right)^2}{\left(1+\sin\frac{\pi}{m}\right)^2}.$$

$$\pi(r-2x)^2=\frac{a\left(1-\sin\frac{\pi}{m}\right)^2}{\left(1+\sin\frac{\pi}{m}\right)^2}=$$

area of each child's farm$=a\left\{\frac{\sin\frac{\pi}{m}}{1+\sin\frac{\pi}{m}}\right\}^2=900\left(\frac{\sin\frac{\pi}{7}}{1+\sin\frac{\pi}{7}}\right)^2=82.4049+$acres.

Area of father's farm$=a\left(\frac{1-\sin\frac{\pi}{m}}{1+\sin\frac{\pi}{m}}\right)^2=9\left(\frac{1-\sin\frac{\pi}{7}}{1+\sin\frac{\pi}{7}}\right)^2=140.292-$acres.

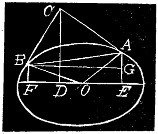
Excellent solutions were received from *P. S. BERG, J. A. CALDERHEAD, LEONARD E. DICKSON, H. C. WHITAKER, J. F. W. SCHEFFER, SETH PRATT, A. H. BELL*, and *J. R. BALDWIN.*

12. Proposed by Professor J. F. W. SCHEFFER, A. M. Hagerstown, Maryland.

Let OA and OB represent two variable conjugate semi-diameters of the ellipse $\frac{y^2}{a^2}+\frac{y^2}{b^2}=1$. On the chord AB as a side described an equilateral triangle ABC. Find the locus of C.

Solution by the PROPOSER.

$OD=x$, $CD=y$, $\angle BOF=\phi$, $\angle AOE=\lambda$, $\angle ABG=\beta$. $y=BC\sin(60^\circ+\beta)+OB\sin\phi=AB\sin(60^\circ+\beta)+OB\sin\phi=AB(\frac{1}{2}\sqrt{3}\cos\beta+\frac{1}{2}\sin\beta)+OB\sin\phi\ldots.(1)$.



$x=OB\cos\phi-BC\sin(60^\circ+\beta)=OB\cos\phi-AB\cos(60^\circ+\beta)=OB\cos\phi-AB(\frac{1}{2}\cos\beta-\frac{1}{2}\sqrt{3}\sin\beta)\ldots.(2)$. But $AB\sin\beta=AE-BF=AO\sin\lambda-OB\sin\phi$, $AB\cos\beta=OE+OF=AO\cos\lambda+OB\cos\phi$.

Substituting these in (1) and (2), we have $y=\frac{1}{2}AO(\sqrt{3}\cos\lambda+\sin\lambda)+\frac{1}{2}OB(\sqrt{3}\cos\phi+\sin\phi)..(3)$.

$x=\frac{1}{2}AO(\sqrt{3}\sin\lambda-\cos\lambda)+\frac{1}{2}OB(\cos\phi-\sqrt{3}\sin\phi)\ldots.(4)$.

But by the well-known relation of conjugate diameters,

$$AO^2=\frac{a^2b^2}{a^2\sin^2\lambda+b^2\cos^2\lambda}=\frac{a^4\sin^2\phi+b^4\cos^2\phi}{a^2\sin^2\phi+b^2\cos^2\phi},\text{ for }\tan\phi\tan\lambda=\frac{b^2}{a^2},$$

$$BO^2=\frac{a^2b^2}{a^2\sin^2\phi+b^2\cos^2\phi},\ \sin\lambda=\frac{b^2\cos\phi}{(a^4\sin^2\phi+b^4\cos^2\phi)^{\frac{1}{2}}},$$

$$\cos\lambda=\frac{a^2\sin\phi}{(a^4\sin^2\phi+b^4\cos^2\phi)^{\frac{1}{2}}}.$$

Substituting in (3) and (4) we obtain after some easy reductions:

$$y=\frac{(a\sqrt{3}+b)(a\sin\phi+b\cos\phi)}{2(a^2\sin^2\phi+b^2\cos^2\phi)^{\frac{1}{2}}} \text{ and } x=\frac{(a+b\sqrt{3})(b\cos\phi-a\sin\phi)}{2(a^2\sin^2\phi+b^2\cos^2\phi)^{\frac{1}{2}}}.$$

Squaring and adding these two equations we have

$$\frac{y^2}{(b+a\sqrt{3})^2}+\frac{x^2}{(a+b\sqrt{3})^2}=\frac{1}{2}$$ or the equation of the required locus.

Also solved in a very excellent manner by *Professors ALFRED HUME* and *G. B. M. ZERR.*

13. Proposed by HENRY HEATON, M. S., Atlantic, Iowa.

Trough two given points to pass four spherical surfaces tangent to two given spheres.

I. Solution by the PROPOSER.

On lines through the external center of similitude and the given points, two other points may be found through which the spherical surface must pass if the given spheres are tangent to it either both externally or both internally.

These points are found in a manner exactly similar to that in which the point U is found in the solution to problem 14. A circle will pass through the four points, which is evidently the circle in which the plane of the two lines through the points cuts the required spherical surface or surfaces.

If a plane be passed through the center of either of the given spheres and that of either of the required spherical surfaces it will pass through the the point of tangency and the section will be great circles of the two spheres tangent to each other. It will cut tne circle through the four points in two other points.

Hence if we pass a plane through the center of either of the given spheres and through the center of the circle through the points and perpendicular to its plane, the section will be a great circle of the given sphere and two points of the circle. Circles through these two points tangent to the circle thus found, will be great circles of two of the required spherical surfaces. The spherical surfaces through these great circles, will be those to which the two given spheres are tangent either both externally or both internally.

The other two spherical surfaces may be found in a manner exactly similar by taking points on lines through the internal center of similitude.

II Solution by H. C. WHITAKER, B. S., M E., Professor of Mathematics, Manual Training School. Philadelphia, Pennsylvania.

Let the equation of the required sphere be

$$(x-l)^2+(y-m)^2+(z-n)^2-R^2=0.$$

Then we have using the first condition

$$(a_1-l)^2+(b_1-m)^2+(c_1-n)^2-(R\pm r_1)^2=0.$$

And also by using the second condition

$$(a_2-l)^2+(b_2-m)^2+(c_2-n)^2-(R\pm r_2)^2=0.$$

And also by using the third condition

$$(x_1-l)^2+(y_2-m)^2+(z_2-n)^2-R^2=0.$$

And also by using the fourth condition

$$(x_2-l)^2+(y_2\quad m)^2+(z_2-n)^2-R^2=0.$$

The required equation is found by eliminating l, m, n and R giving the equation,

$$\left|\begin{array}{lllll} x^2+y^2+z^2 & x & y & z & 1 \\ a_1{}^2+b_1{}^2+c_1{}^2 & a_1 & b_1 & c_1 & (1\pm r_1)^2 \\ a_2{}^2+b_2{}^2+c_2{}^2 & a_2 & b_2 & c_2 & (1\pm r_2)^2 \\ x_1{}^2+y_1{}^2+z_1{}^2 & x_1 & y_1 & z_1 & 1 \\ x_2{}^2+y_2{}^2+z_2{}^2 & x^2 & y^2 & z^2 & 1. \end{array}\right\} =0.$$

CALCULUS

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

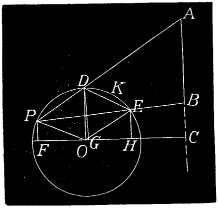
SOLUTIONS TO PROBLEMS.

10. Proposed by ERIC DOOLITTLE, Instructor in Mathematics, State University of Iowa.

Prove or disprove the following theorem: If O be any circle, and AB any straight line either within or without the circumference, and if a perpendicular be dropped from O upon AB and prolonged backward to meet the circumference in P, *then* will the angle whose vertex lies at P and whose sides pass through A and B, cut a portion CPD from the circle which shall be greater than that cut by any other angle whose vertex lies on the circumference and whose sides pass through A and B.

Solution by G. B. M. ZERR, Principal of High School, Staunton, Virginia.

Let O be the centre of the given circle A, B the given points in the line ABC, OC the perpendicular from O on ABC, P any point in the circumference of the circle, D the point where AP cuts the circumference, E the point where PB cuts the circumference.



Let (x_1, y_1) be the coordinates of P, and $(a, b,)$ and (a, c) the coordinates of A, and B; O the origin then $x^2+y^2=r^2$ is the equation to the circle, also $x_1{}^2+y_1{}^2=r^2$.

Equation to PA is $(a-x_1)y+(y_1-b)x=ay_1-bx_1 \ldots . (2)$.

Equation to PB is $(a-x_1)y+(y_1-c)x=ay_1-cx_1 \ldots . (3)$.

Eliminating x between $x^2+y^2=r^2$ and (2) we get $y^2+\dfrac{2y(a-x_1)(bx_1-ay_1)}{(a-x_1)^2+(b-y_1)^2}=\dfrac{r^2(b-y_1)^2-(bx_1-ay_1)^2}{(a-x_1)^2+(b-y_1)^2}$.

$$\therefore \; y+\frac{(a-x_1)(bx_1-ay_1)}{(a-x_1)^2+(b-y)^2}=\pm\frac{(b-y_1)\sqrt{r^2(b-y_2)^2+r^2(a-x_1)^2-(bx_1-ay_1)^2}}{(a-x_1)^2+(b-y_1)^2}$$

but $r^2(b-y_1)^2+r^2(a-x_1)^2=a^2r^2+b^2r^2+r^4-2r^2(ax_1+by_1)$

and $(bx_1-ay_1)^2=-(ax_1+by_1)^2+a^2r^2+b^2r^2$.

$$\therefore \quad y+\frac{(a-x_1)(bx_1-ay_1)}{(a-x_1)^2+(b-y_1)^2}=\pm\frac{(b-y_1)(r^2-ax_1-by_1)}{(a-x_1)^2+(b-y_1)^2}.$$

$$\therefore y=\frac{2br^2+(a^2-b^2-r^2)y_1-2abx_1}{(a-x_1)^2+(b-y_1)^2} \text{ and } y=y_1.$$

$$x=\frac{2ar^2+(b^2-a^2-r^2)x_1-2aby_1}{(a-x_1)^2+(b-y_1)^2} \text{ and } x=x_1.$$

Similarly, $y=\dfrac{2cr^2+(c^2-a^2-r^2)y_1-2acx_1}{(a-x_1)^2+(c-y_1)^2}$ and $y=y_1$.

$$x=\frac{2ar^2+(c^2-a^2-r^2)x_1-2acy_1}{(a-x_1)^2+(c-y_1)^2} \text{ and } x=x_1.$$

Let $\angle COP=\theta$, $\angle COE=\phi$, $\angle COD=\psi$.

Area $PDKE$=area triangle PDE+area segment DKE.

Area $=\dfrac{r^2}{2}[\sin\theta\,(\cos\psi-\cos\phi)+\sin\phi\,(\cos\theta-\cos\psi)+\sin\psi\,(\cos\phi$

$-\cos\theta)]+\dfrac{r^2}{2}[(\psi-\phi)-\sin(\psi-\phi)]=\dfrac{r^2}{2}[\sin\theta\cos\psi-\sin\theta\cos\phi+\sin\phi\cos\theta$

$-\sin\psi\cos\theta+\psi-\phi]=\dfrac{r^2}{2}[\sin(\theta-\psi)+\sin(\phi-\theta)+\psi-\phi]=$maximum.

$$\operatorname{Sin}\theta=\frac{y'}{r},\ \cos\theta=-\frac{x'}{r},\ \sin\phi=\frac{2cr^2+(a^2-c^2-r^2)y_1-2acx_1}{r(a-x_1)^2+r(c-y_1)^2},$$

$$\cos\theta=\frac{2ar^2+(c^2-a^2-r^2)x_1-2acy_1}{r(a-x_1)^2+r(c-y_1)^2},\ \sin\psi=\frac{2br^2+(a^2-b^2-r^2)y_1-2abx_1}{r(a-x_1)^2+r(b-y_1)^2},$$

$$\cos\psi=\frac{2ar^2+(b^2-a^2-r^2)x_1-2aby_1}{r(a-x_1)^2+r(b-y_1)^2}.$$

$$\therefore \quad \text{Area}=\frac{r^2}{2}\Bigg[\frac{2ar^2y_1+(b^2-a^2-r^2)x_1y_1-2aby_1^2}{r^2(a-x_1)^2+r^2(b-y_1)^2}$$

$$-\frac{2ar^2y_1+(c^2-a^2-r^2)x_1y_1-2acy_1^2}{r^2(a-x_1)^2+r^2(c-y_1)^2}-\frac{2cr^2x_1+(a^2-c^2-r^2)x_1y_1-2acx_1^2}{r^2(a-x_1)^2+r^2(c-y_1)^2}$$

$$+\frac{2br^2x_1+(a^2-b^2-r^2)x_1y_1-2abx_1^2}{r^2(a-x_1)^2+r^2(b-y_1)^2}+\cos^{-1}\left\{\frac{2ar^2+(b^2-a^2-r^2)x_1-2aby_1}{r(a-x_1)^2+r(b-y_1)^2}\right\}$$

$$-\cos^{-1}\left\{\frac{2ar^2+(c^2-a^2-r^2)x_1-2acy_1}{r(a-x_1)^2+r(c-y_1)^2}\right\}\Bigg].$$

This reduces to

$$\text{Area}=\frac{r^2}{2}\Bigg[\frac{2(a-x_1)(y_1-b)}{(a-x_1)^2+(b-y_1)^2}-\frac{2(a-x_1)(y_1-c)}{(a-x_1)^2+(c-y_1)^2}$$

$$+\cos^{-1}\left\{\frac{2ar^2+(b^2-a^2-r^2)x_1-2aby_1}{r(a-x_1)^2+r(b-y_1)^2}\right\}$$

$$-\cos^{-1}\left\{\frac{2ar^2+c(c^2-a^2-r^2)x_1-2acy_1}{r(a-x_1)^2+r(c-y_1)^2}\right\}\Bigg].$$

Differentrating this expression and also $x_1{}^2+y_1{}^2=r^2$ and eliminating dx_1, dy_1, we get

$$2(x_1-y_1)\left\{\frac{a+b-x_1-y_1}{[(a-x_1)^2+(b-y_1)^2]^2}-\frac{a+c-x_1-y_1}{[(a-x_1)^2+(c-y_1)^2]^2}\right\}$$
$$-\frac{2b(r-a)x_1-[b^2-(a-r^2)]y_1}{[r(a-x_1)^2+r(b-y_1)^2][2br^2+(a^2-b^2-r^2)y_1-2abx_1]}$$
$$+\frac{2c(r-a)x_1-[c^2-(a-r)^2]y_1}{[r(a-x_1)^2+r(c-y_1)^2][2cr^2+(a^2-c^2-r^2)y_1-2acx_1]}=0.$$

From this expression the values of x_1, y_1 that give a maximum can be determined.

11. Proposed by H. W. DRAUGHON, Clinton, Louisiana.

A ribbon, 1 inch wide is wrapped spirally around a right prism, altitude 10 ft., bases of n sides, radius of circumscribed circle, 1 ft., so as to cover the entire convex surface. (1) What is the length of the ribbon? (2) If the ribbon is unwound and kept tense, by a power acting on the lower end, and moving in the plane of the lower base, what will be the length of the curve described by the power?

Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let $ABCD\ldots$ be the projection of the base of the prism on the plane of the paper, O its centre, OA the radius of the circumscribed circle $\angle BOA=\dfrac{2\pi}{n}$.

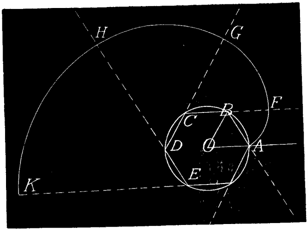
$$\angle ABF=\angle BCG=\&c.=\frac{2\pi}{n},\ AB=2r\sin\frac{\pi}{n}\text{ where }r=OA.$$

(1.) $\therefore$ perimeter of base of prism $=2rn\sin\dfrac{\pi}{n}$.

Now since the ribbon crosses each edge at an inch apart, the length for one revolution $=\sqrt{4r^2n^2\sin^2\dfrac{\pi}{n}+\dfrac{1}{144}}$ feet.

There are in all 120 revolutions, hence,

$$\text{length }=120\sqrt{4r^2n^2\sin^2\frac{\pi}{n}+\frac{1}{144}}=10\sqrt{576n^2\sin^2\frac{\pi}{n}+1}.\quad\text{Since }r=1,$$



when $n=6$, $l=10\sqrt{(576\times9+1)}=720.07$ feet, and when $n=$ infinity, $n\sin\dfrac{\pi}{n}=\pi$.

$\therefore$ $l=10\sqrt{(576\,\pi^2+1)}=754.05$ feet.

(2) Let m be the number of revolutions. Also let $AFGHK\ldots$ be the curve described.

From A to F, $AB=2r\sin\dfrac{\pi}{n}$ is the radius and $\dfrac{2\pi}{n}$ the central angle, from F to

G, $FC=2BC=4r\sin\frac{\pi}{n}$ is the radius and $\frac{2\pi}{n}$ the central angle, from G to H, $GD=3CD=6r\sin\frac{\pi}{n}$ is the radius and $\frac{2\pi}{n}$ the central angle and so on. For the nth side or the last side of the first revolution, radius $=2nr\sin\frac{\pi}{n}$, for the last side of second revolution radius $=4nr\sin\frac{\pi}{n}$, for the last side of the mth revolution radius $=2mnr\sin\frac{\pi}{n}$.

Hence without the use of calculus if $s=$ length of path, arc $AF=2r\times\frac{2\pi}{n}\sin\frac{\pi}{n}$, arc $FG=4r\times\frac{2\pi}{n}\sin\frac{\pi}{n}$, arc $GH=6r\times\frac{2\pi}{n}\sin\frac{\pi}{n}$.

$\therefore\ s=2r\times\frac{2\pi}{n}\sin\frac{\pi}{n}\{1+2+3+\ldots\ (mn-1)\}=2r\times\frac{2\pi}{n}\times\sin\frac{\pi}{n}\frac{mn(mn-1)}{2}$

$=2\pi rm(mn-1)\sin\frac{\pi}{n}$. But $r=1$, $m=120$.

$\therefore\ s=240\pi(120n-1)\sin\frac{\pi}{n}$.

If $n=6$, $s=271057.248$ feet, if $n=$infinity $s=2(120)^2\pi^2=284245.9361$ feet.

Also solved by *ALFRED HUME* and the *PROPOSER*.

12. Proposed by ISAAC L. BEVERAGE, Monterey, Virginia.

Given the equations $2z^3=x+3z$ and $5z^2=y+2z$. To find $\frac{dy}{dx}$ for $x=0$.

Solution by Professor P. H. PHILBRICK, Lake Charles, Louisiana.

We have $x=2z^3-3z$, and $y=5z^2-2z$. $\therefore\ dx=(6z^2-3)dz$, $dy=(10z-2)dz$, and $\frac{dy}{dx}=\frac{10z-2}{6z^2-3}$.

But $x=2z^3-3z=z(2z^2-3)=0$. $\therefore\ z=0$, and $\pm\sqrt{\frac{3}{2}}$.

$\therefore\ \frac{dy}{dx}=\frac{2}{3}$, or $\pm\frac{5\surd6-2}{6}$, when $x=0$.

Also solved by *Professors BLACK, DRAUGHON, MATZ, SCHEFFER, WHITAKER*, and *ZERR*.

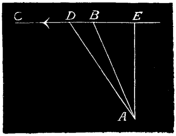
13. Proposed by J. A. CALDERHEAD, Superintendent of Schools, Limaville, Ohio.

A steamer whose curse is due west and speed 10 knots is sighted by another steamer going at 8 knots; what course must the latter steer, so as to cross the track of the former at the least possible distance from her?

Solution by C. W. M. BLACK, A. M., Department of Mathematics. Wilmington Conference Academy, Dover, Delaware.

Let A be the position ot sighting steamer, D of other when sighted, B where A crosses course of D, which by that time has reached C. AE is $CE\perp$. Let $y=CB=CD+DB=CD+DE-BE$. Let $a=$no. hrs. A takes to reach B, and $b=AE$. $\angle EAB=x$, $\angle EAD=\theta$.

$AE'=AB\cos x$, or $b=8a\cos x$, $a=\dfrac{b}{8\cos x}$. $CD=10a$.



$DE=b\tan\theta$, $BE=b\tan x$. $y=\dfrac{10b}{8\cos x}+b\tan\theta-b\tan x$. $\dfrac{dy}{dx}=\dfrac{5b\sin x}{4\cos^2 x}-b\sec^2 x$.

Let $\dfrac{dy}{dx}=0$, whence $\sin\theta=\dfrac{4}{5}$, $\theta=53^\circ 8'$.

The equation shows that b and θ do not affect the result.

Also solved by *Professors PHILBRICK, WHITAKER, ZERR*, and the *PROPOSER*.

14. Proposed by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

Right triangles are inscribed in a circle whose center $=(a, b)$, and radius$=c$. If one of the legs passes through a fixed point, prove that $c^2(x^2+y^2)=(a^2+b^2-c^2-ax-by)^2$ is the curve to which the other leg is always tangent; the fixed point being the origin of the co-ordinates.

Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Let $lx+my=1\ldots.(1)$ be the chord of the circle whose envelope is to be found. Then $y=\dfrac{m}{l}x\ldots.(2)$ is the side of the triangle passing through the fixed point, and $(x-a)^2+(y-b)^2=c^2\ldots.(3)$ is the equation to the circle.

(1) and (2) intersect in $x'=\dfrac{l}{l^2+m^2}$, $y'=\dfrac{m}{l^2+m^2}$.

This point being on (3), we must have on substituting x' and y' in (3), after reducing, $(a^2+b^2-c^2)(l^2+m^2)-2al-2bm+1=0\ldots.(4)$. Making (4) homogeneous in l and m by means of (1) and arranging,

$$[x^2-2ax+(a^2+b^2-c^2)]\frac{l^2}{m^2}+2(xy-bx-ay)\frac{l}{m}+[y^2-2by+(a^2+b^2-c^2)]=0\ldots(5),$$

a quadratic in the parameter $\dfrac{l}{m}$, giving the envelope

$$c^2(x^2+y^2)=[(a^2+b^2-c^2)-by-ax]^2\ldots.(6).$$

Also solved by *P. S. BERG, G. B. M. ZERR* and the *PROPOSER*.

15. Proposed by CHARLES E. MYERS, Canton, Ohio.

From a given quantity of material a cylindrical cup with circular bottom and open top is to be made, the cup to contain the greatest amount. What must be its dimensions?

Solution by F. P. MATZ, M. S., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Represent the radius of the base by x, and the altitude by y; then, obviously, $\pi x^2+2\pi xy=s$. $\therefore\ y=(s-\pi x^2)\div 2\pi x$.

Also, $V=\pi x^2 y=\pi x^2\left(\dfrac{s-\pi x^2}{2\pi x}\right)$, $=$a maximum.

$$\therefore \frac{dV}{dx}=s-3\pi x^2=0. \quad \therefore \quad x=\sqrt{\left(\frac{s}{3\pi}\right)},=y.$$

Also solved by *P. S. BERG, C. W. M. BLACK, J. H. DRUMMOND, A. HUME, P H. PHILBRICK, H. C. WHITAKER, G. B. M. ZERR*, and the *PROPOSER*.

PROBLEMS.

20. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

$$\int_0^{\frac{1}{2}\pi}\sqrt{[(1-e^2\cos^2\phi)(1-e^2\sin^2\phi)]}d\phi=\text{what?}$$

21 Proposed by T. JOHN COLE, Columbus, Ohio.

In the equilateral triangle ABC, AB the base is 10 feet. With B as a center an arc is drawn from C to A; likewise with A as a center an arc is drawn from C to B. What is the volume of the solid generated by revolving the figure about the altitude of the triangle as an axis.

Solutions to these problems should be received on or before August 1st.

MECHANICS.

Conducted by B.F.FINKEL,Kidder, Missouri. All Contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

6. Proposed by THOMAS W. WRIGHT, M. A., Ph. D., Professor of Applied Mathematics and Physics, Union College, Schenectady, New York.

What is the effect of a charge between light and heavy cavalry, the light cavalry having the greater energy and the heavy the greater momentum?

Solution by P. H. PHILBRICK. C. E., Lake Charles, Louisiana.

Let M and V represent the mass and velocety repectively of the heavy cavalry and M_1 and V_1 the same of the light cavalry. Supposing the bodies to be inelastic and moving in opposite directions before impact and together after impact their common velocity after impact is, $V=\dfrac{MV-M_1V_1}{M+M_1}$ ....(1).

Since $MV>M_1V_1$ the heavy cavalry will overcome the lighter and carry it along at the above rate. The combined energy of the bodies is, $\frac{1}{2}MV^2+\frac{1}{2}M_1V_1^2$....(2). This measures the distructive effect of the charge.

7. Proposed by DE VOLSON WOOD, M. A., M. Sc., C. E., Professor of Engineering, Stevens Institute of Technology, Hoboken, New Jersey.

A hollow sphere filled with frictionless water rolls down a rough plane whose length is l and inclination θ; when half way down the water suddenly freezes and adheres to the sphere. Required the time of the decent.

I. **Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics in the University of Mississippi.**

Let M=the mass of the hollow sphere;

m=the mass of the water;

R=the radius of the outer surface of the sphere;

r=the radius of the inner surface of the sphere;

K=the radius of gyration of the shell about a diameter;

k=the radius of gyration of the water about a diameter;

l=the length of the inclined plane;

α=the inclination of inclined plane;

x=the distance from the upper end of the plane to the point where the sphere touches it after t seconds;

θ=the angle turned through;

F=the friction acting upon the plane.

The equations of motion are

$$(M+m)\frac{d^2x}{dt^2}=(M+m)g.\sin\alpha-F\ldots.(1).$$

and, since the water, being frictionless, does not partake of the rotary motion of the shell,

$$MK^2\frac{d^2\theta}{dt^2}=FR\ldots.(2).$$

Also, the plane being perfectly rough, there is no sliding. Hence,

$$x=R\theta,\text{ and }\frac{d^2x}{dt^2}=R\frac{d^2\theta}{dt^2}\ldots.(3).$$

From equations (1), (2), and (3)

$$\frac{d^2x}{dt^2}=\frac{(M+m)R^2g\sin\alpha}{(M+m)R^2+MK^2}.$$

This being the acceleration, the space passed over from rest in t seconds is given by

$$x=\tfrac{1}{2}\frac{(M+m)R^2g\sin\alpha}{(M+m)R^2+MK^2}t^2.$$

The time required to traverse the upper half of the plane is, therefore,

$$\sqrt{\frac{(M+m)R^2+MK^2}{(M+m)R^2g\sin\alpha}l}.$$

and the velocity acquired, v, is

$$\sqrt{\frac{M(+m)R^2g\sin\alpha}{(M+m)R^2+MK^2}l}.$$

Now let

w=angular velocity immediately before the water freezes;

w'=angular velocity immediately after the water freezes;

v'=velocity of center immediately after the water freezes;

Since any change in the motion is due to an impulse at the point of tangency of the sphere with the plane, the angular momentum about a hori-

zontal line in the plane and through this point is not altered. Therefore

$$(M+m)v.R+MK^2.w=(M+m)v'.R+(MK^2+mk^2)w'.$$

Then, since $v=Rw$ and $v'=Rw'$,

$$v'=\frac{(M+m)R^2+MK^2}{(M+m)R^2+MK^2+mk^2}v$$

$$=\frac{\sqrt{[(M+m)R^2+MK^2](M+m)R^2\,g\,\sin\alpha.l}}{(M+m)R^2+MK^2+mk^2}.$$

For the motion down the lower half of the plane the equations are,

$$(M+m)\frac{d^2x}{dt^2}=(M+m)g\sin\alpha-F'$$

and $(MK^2+mk^2)\dfrac{d^2\theta}{dt^2}=F'R$ where F' is the friction.

$$\text{Also, } x=R\theta,\ \frac{d^2x}{dt^2}=R\frac{d^2\theta}{dt^2}.$$

Solving these for t, remembering that when $t=0$, $\dfrac{dx}{dt}=v'$, the time is found to be

$$-\sqrt{\frac{(M+m)R^2+MK^2}{(M+m)R^2g\sin\alpha}l}+\sqrt{\frac{(M+m)R^2+MK^2+(M+m)R^2+MK^2+mk^2}{(M+m)R^2g\sin\alpha}l}.$$

The whole time, therefore, is

$$\sqrt{\frac{2(M+m)R^2+2MK^2+mk^2}{(M+m)R^2g\sin\alpha}l}.$$

Since $K^2=\dfrac{2}{5}\cdot\dfrac{R^5-r^5}{R^3-r^3}$ and $k^2=\dfrac{2}{5}r^2$, this becomes

$$\sqrt{\frac{10(M+m)(R^3-r^3)R^2+4M(R^5-r^5)+2m(R^3-r^3)r^2}{5(M+m)(R^3-r^3)R^2g\sin\alpha}l}.$$

Also solved by *P. H. PHILBRICK, WILLIAM HOOVER*, and *J. C. NAGLE*. We will publish one or all of these excellent solutions in the next issue.

PROBLEMS.

13. Proposed by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

A man, horse and buggy are going around a circular race course at a 2:40 gait. If the whole outfit weighs 1500 lbs, radius of course is 500 feet and track is inclined so that pressure is equal upon the wheels, find the pressure on the ground due to whole weight.

14. Proposed by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University P. O.

"The center of a sphere of radius c moves in a circle of radius a and generates thereby a solid ring, as an anchor ring; prove that the moment of inertia of this ring about an axis passing through the center of the direct circle and perpendicular to its plane is $\dfrac{\pi^2\rho ac^2}{4}(4a^2+3c^2)$."

15. **Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

Show that the *eastward deviation* of bodies falling from a great height is

$$E_d=\frac{4\pi t(H-\frac{1}{2}\triangle)\cos\phi}{3T}.$$

Solutions to these problems should be received on or before August 1st.

DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

5. **Proposed by ISAAC L. BEVERAGE, Monterey, Virginia.**

Find three numbers the sum of the squares of any two of which diminished by their product shall be a square number.

Solution by ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Let rx, ry, rz represent the numbers. Then we must satisfy

$$x^2-xy+y^2=\square\ \ldots.(1),$$
$$y^2-yz+z^2=\square\ \ldots.(2),$$
$$x^2-xz+z^2=\square\ \ldots.(3),$$

rejecting the square factor r^2.

Assume $x=2pq-q^2$, $y=p^2-q^2$, and (1) is satisfied. If we take $p=3$, $q=1$, we have $x=5$, $y=8$, and by substitution (2) and (3) become

$$z^2-5z+25=\square\ \ldots.(4),$$
$$z^2-8z+64=\square\ \ldots.(5).$$

Now put $(5)=(z-2n)^2$ and we get $z=\dfrac{16-n^2}{2-n}$.

Substituting this value of z in (4) and reducing, $n^4-5n^3+3n^2-20n+196=\square=(n^2-\frac{5}{2}n+14)^2$ say, whence $n=\frac{8}{5}$; therefore $z=\frac{168}{5}$, and, taking $r=5$, $rx=25$, $ry=40$, $rz=168$, three numbers satisfying the conditions of the problem,

Also solved by *H. W. DRAUGHON*, and *G. B. M. ZERR.*

6. **Proposed by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.**

Find three whole numbers the sum of any two of which is a cube.

Solution by H. W. DRAUGHON, Clinton, Louisiana.

Let the three numbers be, $\frac{1}{2}(x^3+y^3-z^3)$, $\frac{1}{2}(x^3-y^3+z^3)$, and $\frac{1}{2}(y^3-x^3+z^3)$, then,

$$\tfrac{1}{2}(x^3+y^3-z^3)+\tfrac{1}{2}(z^3-y^3+z^3)=x^3,$$
$$\tfrac{1}{2}(x^3+y^3-z^3)+\tfrac{1}{2}(y^3-x^3+z^3)=y^3, \text{ and}$$
$$\tfrac{1}{2}(y^3-x^3+z^3)+\tfrac{1}{2}(x^3-y^3+z^3)=z^3.$$

In order that the numbers may be positive and integral we must make

$z^3 < x^3+y^3$ and $z^3 > z^3-z^3$, and x, y, and z must be even numbers or two odd and the other even.

Ex. Put $x=9$, $y=7$, and $z=8$, the resulting numbers are, 280, 449, and 63.

Also solved by *P. S. BERG*, *M. A GRUBER*, *ARTEMAS MARTIN*, *H. C. WHITAKER*, and *G. B. M. ZERR*.

8. **Proposed by Hon. JOSIAH H. DRUMMOND, Portland, Maine.**

Every odd square is of the form $4a+1$; find the value of a for the nth consective odd square.

Solution by M. A. GRUBER, A. M., War Department, Washington, D. C., and R H. YOUNG, West Sunbury, Pennsylvania.

The consecutive odd squares are the squares of the consecutive odd numbers.

The difference between two consecutive odd numbers is 2.

Beginning with the odd number 1, the next odd number is 1×2 greater than 1; the 3d odd number is 2×2 greater than 1; the 4th odd number is 3×2 greater than 1, and so on to the nth odd number which is accordingly $n-1$ times 2 greater than 1.

The nth odd number is, therefore, $1+2n-2$, or $2n-1$.

$\therefore$ $(2n-1)^2=4a+1$, and $a=n^2-n=n(n-1)$.

Also solved by *A. H. BELL*, *C. W. M. BLACK*, *H. W. DRAUGHON*, *ARTEMAS MARTIN*, *P. H. PHILBRICK*, *H. C. WHITAKER*, *G. B. M. ZERR*, and the *PROPOSER*

AVERAGE AND PROBABILITY.

Conducted by B.F.FINKEL,Kidder, Missouri. All Contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

Proposed by Miss LEOTA MILLER, B. L., Professor of Natural Science and Art, Kidder Institute, Kidder, Missouri.

A deer, wounded at the corner of a square park, is equally liable to run in a straight line in any direction, from the corner of the park, and, at the same time, is also equally liable to drop dead before running a distance equal to the diagonal of the park. What is the chance that the deer will drop dead in the park?

II. **Solution by W. B MILWARD, Amity, Missouri, and P. H. PHILBRICK, C. E., Lake Charles, Louisiana.**

Let $ABCD$ represent the park diameter a, and describe a circle with center A and radius$=a\surd2=AC$ the diagonal of the park. Area of park$=a^2$; area of circle$=\pi(a\surd2)^2=2\pi a^2$. The area of the circle represents one half of all possible ground upon which the deer will fall. Hence the required probability is $\dfrac{a^2}{4\pi a^2}=\dfrac{1}{4\pi}$.

[REMARK:—Professor Philbrick writes, June 21: It [the problem above] is

misquoted in the May Number. The word "equally" before liable is omitted. Not noting that word probably accounts for the different solutions."

Professor Philbrick understands the problem to mean that the deer is just as liable to drop dead before running a distance equal to the diagonal of the park as it is to run a greater distance.

This is the meaning we get from the statement of the problem. Now the solutions printed in May No. assume that the deer actually drops dead within the circle whose radius is the diagonal of the square, while the above solution assumes that the probability that deer drops dead within this circle is $\frac{1}{2}$.

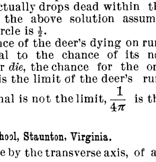
We take "equally liable" to mean that the chance of the deer's dying on running a distance equal to the diagonal of the park is equal to the chance of its not dying on running that distance, and since it must *live* or *die*, the chance for the one or the other is $\frac{1}{2}$. Hence, if the diagonal of the square is the limit of the deer's running before death ensues, the answer is $\frac{1}{2\pi}$; if the diagonal is not the limit, $\frac{1}{4\pi}$ is the answer. Ed.]

4. **Proposed by Professor G. B. M. ZERR, Principal of High School, Staunton, Virginia.**

Four points taken at random in each half, made by the transverse axis, of an ellipse, are joined in such a way by straight lines as to inclose an octagonal surface; find the mean area of this surface.

Solution by the PROPOSER.

Let P, Q, R, S be four points in the semi-ellipse, axis a, b, above the transverse axis AA', D, E, F, G four points in the lower half. Through these draw $P'K$, $Q'N$, $R'T$, $S'B$, $D'L$, $E'M$, $F'H$, $G'C$ perpendicular to AA'. Let $\angle AOP'=\theta$, $\angle AOQ'=\phi$, $\angle AOR'=\psi$, $\angle AOS'=\rho$, $\angle AOD'=\mu$, $\angle AOE'=\nu$, $\angle AOF'=\omega$, $\angle AOG'=\lambda$, $KP=x$, $NQ=w$, $SB=z$, $DL=s$, $RT=y$, $ME=t$, $HF=u$, $CG=v$. Then we have $KP''=b\sin\theta=x'$, $NQ''=b\sin\phi=w'$, $TR''=b\sin\psi=y'$, $BS''=b\sin\rho=z'$, $LD''=b\sin\mu=s'$, $ME''=b\sin\nu=t'$, $HF''=b\sin\omega=u'$, $CG''=b\sin\lambda=v'$. Area of the octagonal surface equals the area of the six triangles $RGS+RGF+FRQ+QFE+EQP+PED$.



$\therefore A=\text{area}=\frac{1}{2}a\{x(\cos\phi-\cos\mu)+w(\cos\psi-\cos\theta)+y(\cos\rho-\cos\phi)+z(\cos\lambda-\cos\psi)+s(\cos\nu-\cos\theta)+t(\cos\omega-\cos\mu)+u(\cos\lambda-\cos\nu)+v(\cos\rho-\cos\omega)\}$.

An element of surface at P is $a\sin\theta\, d\theta dx$; at Q, $a\sin\phi d\phi dw$; at R, $a\sin\psi d\psi dy$; at S, $a\sin\rho d\rho dz$; at D, $a\sin\mu d\mu ds$; at E, $a\sin\nu d\nu dt$; at F, $a\sin\omega d\omega d\omega$; at G, $a\sin\lambda d\lambda dv$.

The limits of θ and μ are 0 and π; of ϕ, 0 and θ; of ψ, 0 and ϕ; of ρ, 0 and ψ; of ν, 0 and μ; of ω, 0 and ν; of λ, 0 and ω. Hence, the reqired average area is,

$$\Delta=\frac{\int_0^\pi\int_0^\theta\int_0^\phi\int_0^\psi\int_0^\pi\int_0^\mu\int_0^\nu\int_0^\omega\int_0^{x'}\int_0^{w'}\int_0^{y'}\int_0^{z'}\int_0^{s'}\int_0^{t'}\int_0^{u'}\int_0^{v'} Aa\sin\theta d\theta a\times\sin\phi d\phi a\sin\psi d\psi a\sin\rho d\rho a\sin\mu d\mu a\sin\nu d\nu a\sin\omega d\omega a\sin\lambda d\lambda dx dw dy dz ds dt du dv}{\int_0^\pi\int_0^\theta\int_0^\phi\int_0^\psi\int_0^\pi\int_0^\mu\int_0^\nu\int_0^\omega\int_0^{x'}\int_0^{w'}\int_0^{y'}\int_0^{z'}\int_0^{s'}\int_0^{t'}\int_0^{u'}\int_0^{v'} a\sin\theta d\theta a\sin\phi d\phi a\times\sin\psi d\psi a\sin\rho d\rho a\sin\mu d\mu a\sin\nu d\nu a\sin\omega d\omega a\sin\lambda d\lambda dx dw dy dz ds dt du dv}$$

$$\varDelta=\frac{2^{14}3^2}{\pi^8 b^8}\int_0^\pi\int_0^\theta\int_0^\phi\int_0^\Psi\int_0^\pi\int_0^\mu\int_0^\nu\int_0^\omega\int_0^{x'}\int_0^{w'}\int_0^{y'}\int_0^{z'}\int_0^{s'}\int_0^{t'}\int_0^{u'}\int_0^{v'} A\sin\theta\sin\phi$$
$$\times\sin\psi\sin\rho\sin\mu\sin\nu\sin\omega\sin\lambda\, d\theta d\phi d\psi d\rho d\mu d\nu d\omega d\lambda dx dw dy dz ds dt du dv$$
$$=\frac{2^{12}3^2ab}{\pi^8}\int_0^\pi\int_0^\theta\int_0^\phi\int_0^\Psi\int_0^\pi\int_0^\mu\int_0^\nu\int_0^\omega[\sin(\theta-\phi)+\sin(\phi-\psi)+\sin(\psi-\rho)$$
$$+\sin(\mu-\nu)+\sin(\nu-\omega)+\sin(\omega-\lambda)+\sin(\lambda-\rho)-\sin(\theta+\mu)]\sin^2\theta\sin^2\phi\sin^2\psi\sin^2\rho$$
$$\times\sin^2\mu\sin^2\nu\sin^2\omega\sin^2\lambda\, d\theta d\phi d\psi d\rho d\mu d\nu dd\omega\lambda,$$
$$=\frac{2^6 3ab}{\pi^4}\int_0^\pi\int_0^\theta\int_0^\phi\int_\theta^\Psi[\sin(\theta-\phi)+\sin(\phi-\psi)+\sin(\psi-\rho)]\sin^2\theta\sin^2\phi\sin^2\psi\sin^2\rho$$
$$d\theta d\phi d\psi d\rho \quad +\frac{2^{12}3^2ab}{\pi^8}\int_0^\pi\int_0^\theta\int_0^\phi\int_0^\Phi\left\{\left(\frac{\pi^2}{45}-\frac{43264}{496125}\right)\sin\theta+\left(\frac{\pi^2}{45}-\frac{43264}{496125}\right)\sin\rho\right.$$
$$\left.+\left(\frac{\pi^3}{72}-\frac{352\pi}{4725}\right)\cos\rho-\left(\frac{\pi^3}{72}-\frac{352\pi}{4725}\right)\cos\theta\right\}\sin^2\theta\sin^2\phi$$
$$x\sin^2\psi\sin^2\rho d\theta d\phi d\psi d\rho$$
$$=\frac{ab}{\pi}\left\{5-\frac{7667}{240\pi^2}\right\}+\frac{2^{14}ab}{45\pi^7}\left\{\frac{\pi^2}{8}-\frac{352}{525}\right\}\left\{\pi^2-\frac{43264}{11025}\right\}.$$

A partial solution was received from *Professor F. P. Matz.*

5. **Proposed by DE VOLSON WOOD, M. A., C. E., Professor of Mechanical Engineering, Stevens Institute of Technology, Hoboken, New Jersey.**

An actual case suggested the following:

An equal number of white and black balls of equal size are thrown into a rectangular box, what is the probability that there will be contiguous contact of white balls from one end of the box to the opposite end? As a special example, suppose there are 30 balls in the length of the box, 10 in the width, and 5 (or 10) layers deep.

Solution by Professor P. H. PHILBRICK, M. S., C. E., Lake Charles, Louisiana.

There are $5.10=50$ rows and $50\times30=1500$ balls. The probability will be found by supposing the balls to be drawn singly and at random and the rows formed by the balls as drawn one after another.

The total number of ways of drawing the balls$=\underline{|1500}=a$. To find the number of ways of drawing the balls so that 30 white balls may form a row, 30 white balls may be selected out of 750 in $\dfrac{750.746\ldots.721}{1.2.3\ldots.30}=b$ ways. The remaining 1470 balls can be drawn in $\underline{|1470}=c$ ways. In order that a given selection of 30 white balls may form a row they must be drawn either before or after the other 49 rows, or between some of them, giving 50 positions.

The number of favorable cases is equal to the product of the last three quantities$=50bc$. Hence the probability is, $\dfrac{50bc}{a}$.

Also solved by *Professor W. H. Draughon.*

[Note.—This solution is not entirely satisfactory, since the white balls may be contiguous from one side of the box to the other without being in longitudinal lines. The problem is as if it were required to find the probability of passing from one side of the box to the other by passing through white balls, and white balls only, under the conditions named. The problem is a pretty good one and if any one will

furnish a complete solution to it, we will publish it in the next issue of the MONTHLY. ED.]

PROBLEMS.

9. Proposed by H. C. WHITAKER, B. S., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

Four numbers taken at random are multiplied together. What is the probability that the last digit will be 0?

10. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College. New Windsor, Maryland.

Upon a surface one foot square, a coin one inch in diameter is thrown; what is the chance the coin *touches* or *intersects* both diagonals?

11. Proposed by ARTEMAS MARTIN, A. M., Ph. D., LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Find the average area of a triangle formed by joining a corner of cube with any two points within the cube.

12. Proposed by Professor G. B M. ZERR, A. M., Principal of High School, Staunton, Virginia.

A large plane area is ruled by two sets of parallel eqidistant straight lines, the one set perpendicular to the other. The distance between any two lines of the first set is a; the distance between any two lines of the second set is b. If a regular polygon of $2n$ sides be thrown at random upon this area, find the chance that it will fall across a line, the diameter of the circum-circle of the polygon being less than a or b.

Solutions to these problems should be received on or before August 1st.

MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

7. Proposed by Rev. A. L. GRIDLEY, Pastor of Congregational Church, Kidder, Missouri.

Making no allowance for the curvature of the earth and supposing the sun to rise in the east and set in the west, what would be the course of a man who should walk constantly toward the sun from morning until night? How far and in what direction from the staring point would he be, walking three miles per hour, at the end of three days?

Solution by H. C. WHITAKER, B. S., C. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

Let the azimuth of the sun and of the direction in which the man is walking be counted from the south toward the west and let it be denoted by ϕ. Take as the origin the position of the man at noon and let the time (t) be counted

at noon. Let r denote the rate of the man, s the distance he walks in time t, l his latitude *assumed to be constant*, h the hour angle after noon. Then from spherical trigonometry, $\cot\phi=\sin l\cot h$ (1). But $h=\frac{\pi t}{12}=\frac{\pi s}{12r}$, hence $\cot\phi=\sin l\cot\frac{\pi s}{12r}$ (2) which is the intrinsic equation of the path of the man, latitude assumed constant.

From this, $ds=\frac{12r\sin l}{\pi}\frac{d\phi}{1-\cos^2 l\sin^2\phi}$

$$x=\int\sin\phi ds=\frac{12r\sin l}{\pi}\int\frac{\sin\phi d\phi}{1-\cos^2 l\sin^2\phi}=\frac{12r}{\pi\cos l}\left[\tan^{-1}\frac{\sin l\tan(\frac{\pi}{4}+\frac{\phi}{2})}{1+\cos l}\right.$$

$$\left.-\tan^{-1}\frac{\sin l\tan(\frac{\pi}{4}+\frac{\phi}{2})}{1+\cos l}-\tan^{-1}\frac{\sin l}{1-\cos l}+\tan^{-1}\frac{\sin l}{1+\cos l}\right] \quad (3). \qquad y=\int\cos\phi d\theta$$

$$=\frac{12r\sin l}{\pi}\int\frac{\cos\phi d\phi}{1-\cos^2 l\sin^2\phi}=\frac{6r\sin l}{\pi}\log_e\left[\frac{1+\cos l\sin\phi}{1-\cos l\sin\phi}\right] \quad (4).$$

Since only y is desired, in (2) we let $s=6r$, then $\phi=\frac{\pi}{2}$, whence in (4)

$y=\frac{6r\sin l}{\pi}\log_e\cot^2\frac{1}{2}l=\frac{12r\sin l}{\pi}\log_e\cot\frac{1}{2}l$ which is the distance traveled south in one-half day. In 3 days at 3 miles per hour $y=\frac{216\sin l}{\pi}\log_e\cot\frac{1}{2}l$.

QUERIES AND INFORMATION.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

Answers to Queries in the American Mathematical Monthly for March 1894. (Vol. I. No. 3. page 102).

Continued from the May number.

IV. In the spaces called after Riemann, called by Klein *elliptic*, the whole straight line is finite.

Two such straights, having crossed, recur to the point of crossing without going through any point at infinity.

V. In Euclid's constructions, only pieces of straights occur, each piece having two given end points. Such pieces are *sects*, always finite. But as soon as, with von Staudt, we admit a point at infinity, then we have straights with two ends, yet infinite; for the whole straight is infinite, and so its half is infinite.

One of the two costraight rays from a given point to a point at infinity is always infinite.

VI. In Lobatschewsky's geometry, all coplanar copunctal straights are divided, with reference to a given coplanar straight, into *cutting* and *not-cutting*, by two boundary lines, which do not cut the given line for any finite construction, but each of which has a point at infinity in common with the given line.

VII. Lobatschewsky's parallels are therefore coplanar straights which however far they can actually be produced do not meet, yet which meet at infinity.

In Lobatchewsky's space, two straight lines perpendicular to a third never intersect, how far soever they be produced; yet they are not parallel, for they do not even have a common point at infinity, which is as much an essential of parallelism in Lobatschewsky's non-Euclidean space as in von Staudt's Euclidean space.

GEORGE BRUCE HALSTED.

Remark on Mr. Stevens' Article in April Number.

From equation (2) $\log(-1)=\pi(1+2a)\sqrt{-1}$ we have by dividing, $\pi=\dfrac{\log(-1)}{(2a+1)\sqrt{-1}}$ or making $a=0, \pi=\dfrac{\log(-1)}{\sqrt{-1}}$, a most singular result in Mathematics. This result can also be expressed in the form, $\pi=2\sqrt{(-1)}\log\dfrac{1-\sqrt{(-1)}}{1+\sqrt{(-1)}}$.

COOPER D. SCHMITT.

QUERY.—Is there on the Western Continent a copy of the work of Giordano da Bitonto: Euclide restituto overe gli antichi elementi geometrici ristoranti, Roma, 1680. Folio?

TRANSLATOR OF LOBATSCHEWSKY, GEORGE BRUCE HALSTED.

EDITORIALS.

This issue is mailed a week late.

Dr. Paul Stackel writes from the University of Halle of his interest in the AMERICAN MATHEMATICAL MONTHLY'S Non-Euclidean Geometry.

We shall be very grateful to our subscribers if they will aid us in doubling the number of subscribers to the MONTHLY during July and August.

Subscribers, we shall be pleased to send sample copies of the MONTHLY to any of your friends who are likely to subscribe if you will kindly give us their address.

Remember, we will send the MONTHLY one year to any person sending us 4 names and $8. Send money by Post Office Money Order or by Draft to B. F. Finkel, Kidder, Missouri.

Professor David E. Smith, Ph. D., of the Michigan State Normal School, writes Dr. Halsted as follows: "I am enjoying your papers on Non-Euclidean Geometry in the AMERICAN MATTEMATICAL MONTHLY, as I must say I always enjoy your articles."

Mrs. Eva S. Maglott, A. M., Professor of Mathematics in the Ohio Normal University, writes us that she is well pleased with the MONTHLY and that it is just the kind of a Journal she has been wanting for several years. The Ohio Normal University does thorough work in Mathematics and finds use for such Journals as the MONTHLY.

Six numbers of the AMERICAN MATHEMATICAL MONTHLY have now been issued, containing over 200 pages, and costing each subscriber $1.00.

Has there ever been a Mathematical Journal of its quantity and quality offered at $2.00 a year?

We tender our best thanks to our valued contributors for the interest they have manifested in the first half years existence of the MONTHLY, and hope they will continue with us as long as the MONTHLY is published.

Professor Wm. Symmonds, Professor of Mathematics and Astronomy, Pacific College, Santa Rosa, California, writes: "Consider me a constant subscriber for the MONTHLY."

The President of the physico-mathematical Society of Kasan has made a printed report in German and French of the state of the Lobatschewsky-Foundation. Between the 22d of February, 1893, and February, twenty-second, 1894, were received 7086 Roubles and 38½ Copeks (about 15748 Marks or 19684 Francs). Of this, 6019 roubles 10½ copeks come from Russia, and of this 2536 roubles come from Kasan. Germany gives 201 roubles 44 copeks, the United States gives 174 roubles 11 copeks, France, 157 roubles 80 copeks, and England, 85 roubles 87 copeks. In Russia more than 800 persons subscribed. From other lands more than 150. Besides these individuals, part in the subscription has been taken by 1. The Royal Society of London, 2. The Mathematical congress at Chicago [Subscription paper passed by Dr. Macfarlane after a Lecture by Dr. Halsted],

3. Les membres de la Faculte des Sciences de Paris,
4. Les membres de la Faculte des Sciences de Nancy,
5. The Mathematical Society of Goettingen,
6. The Mathematical Society of Amsterdam,
7. Der Verband alter Herren des mathematichen Vereins der Universitaet Berlin,
8. La direction de la "Revue de metaphysique et de morale,"
9. La direction de l' "Intermedrare des mathematiciens."
10. The Kansan government Council,
11. The Mathematical Society of Charkow,
12. The cities of Nischni, Samara, and Omsk.

Since the subscriptions continue to flow in, in great part because of the activity of the *Ehrenmitglieder der Comites*, the state of the capital will certainly permit not only of founding a prize worthy of the name of Labatchewsky, (e. g. a triennial prize of 500 roubles) but also of placing a bust of the great geometer in the Park bearing his name in front of the University to which he consecrated all his noble life.

La Real Academia de Ciencias Exactas, Fisicas y Naturales de Madrid offer First, and Second Prizes, and Honorable Mention for theses on the following subject: *Didactic treatment of the modern non-Euclidean geometric theories, or reasoned analysis of the principal works on this part of mathematical science.* The First prize consists in a special Diploma, a gold medal of 60 grammes weight, also one thousand five hundred pesetas in money, and the printing of the thesis at the cost of the Academy, and the delivery to the author of one hundred copies. The second prize is the same, except the money. Theses will be received until December 31st, 1895.

BOOKS.

The Elements of Geometry. Revised Edition, 1894. By Webster Wells, S. B., Professor of Mathematics in the Massachusetts Institute of Technology. Half morocco. XII+3?2 pp. Price, $1.25. Boston: Leach, Shewell, and Sanborne.

Professor Wells has fully met the wants of schools of lower grade as well as the more advanced requirements of colleges and scientific schools. A good feature of the book is that, in reading a demonstration, a page does not need to be turned in order to refer to the figure. A special feature of the work consists in its numerous easy exercises and numerical problems. The book is finding its way into the best schools and colleges in the United States, which is a splendid recommendation for a text so recently published. B. F. F.

First Lessons in our Country's History. Revised Edition. By William Swinton. Price 48cts. New York: American Book Co.

This popular little book of the late Professor Swinton aims to bring out to prominent view the salient points of our Country's history, and such only, and to combine simplicity with sense in the mode of presentation. Revised, enlarged, illustrated anew, and brought down to date, it is believed the present edition will prove even more acceptable than the earlier edition. J. M. C.

A First Book in Algebra. By Wallace C. Boyden, A. M., Sub Master of the Boston Normal School. 176, pp. Price 60cts. Boston: Silver, Burdett & Co. 1894.

This book is designed to meet the demand for a text-book in Algebra suited to the needs of the lower grades. The points of difference between Algebra and Arithmetic are carefully explained, and instead of making a statement of what the pupil is to see in the illustrative example, questions are asked which will lead him to find out what he is to discover from it. We note with pleasure the abundant practice given the representation of numbers by letters and the great care taken to make clear the meaning of the minus sign, as applied to a simple number together with the modes of operating upon negative numbers. The book is well supplied with examples and strikes us as thoroughly adapted to the purpose for which it is designed J.M.C.,

Geometry in The Grammar School: An Essay. Together with illustrated class exercises, and an Outline of the Work for the Last Three Years of the Grammar School. By Paul N Hanus, Assistant Professor of the History and Art of Teaching, Harvard University. 8vo, paper back, 52 pp. Price, $0.25. Boston: D. C. Heath & Co.

The unexperienced teacher of geometry can receive no better aid than that received from a careful reading and study of this essay. Even the experienced teacher of geometry may find in it much of interest to him. B. F. F.

Plane Trigonometry. Part I. An Elementary Course, Excluding the use of Imaginary Quantities. By L. L. Loney, M. A., Late fellow of Sidney Sussex College, Cambridge, Professor at the Royal Holloway College. 8vo cloth, XX+292 pp. Price, $1.40. New York: Macmillan & Co.

This book consists of the more elementary portions of a text-book on Trigonometry. It covers the whole course usually read in Schools,and excludes De Moivre's Theorem and the Higher Analytical Trigonometry. *Preface.*

The book contains twenty chapters under which are treated every subject within the purview of Plane Trigonometry and 750 well selected problems. The typography of the work is first class. We made mention of the second part of Professor Loney's Trigonometry in the May Number of the MONTHLY. If the author writes a third part on Spherical Trigonometry, the three parts will make the most complete treatise on Trigonometry with which we are acquainted. B. F. F.